

# An Algorithm for Detecting P, QRS, and T Waves in a Single Lead ECG

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## Abstract

An algorithm for detecting the onsets, offsets and peaks of P, QRS and T waves in a single lead ECG is presented. The algorithm is designed to be effective for all standard leads with various sampling rates.

## 1 Introduction

For the purposes of this discussion, a single lead ECG is composed of a sequence of measurements called *samples*. Samples are integers which map to discrete millivolt measurements. A typical sample mapping is  $[-2^{11}, 2^{11}] \rightarrow [-10mV, 10mV]$ . The number of discrete samples per millivolt is the *sampling resolution*. Samples are recorded at regular intervals in time. The rate at which samples are taken is called the *sampling rate*. Typical sampling rates are 125Hz, 250Hz, 360Hz, or 1000Hz.

The three fundamental features of each heart beat in the ECG are the P wave which represents atrial depolarization, the QRS complex which represents ventricular depolarization, and the T wave which represents ventricular repolarization [5]. The proposed algorithm detects the onsets, offsets, and peaks of these three features. The onset and offset of the QRS complex is defined as the onset of the Q wave and the offset of the S wave respectively.

Various methods have been applied to identify P, QRS, and T waves in the ECG. Okada [6] presents a digital filter for enhancing the QRS complex. Pan

and Thompkins [7] use a similar method with a moving window integrator to detect QRS complexes. Reddy et al. [4] use a derivative filter and thresholds to detect the P wave. Laguna et al. [3] use the signal slope for the identification of the QRS wave, and they use derivatives to find the peaks of the P and T waves. Koski et al. [2] use syntactic methods based on signal slope to identify the P, QRS, and T waves.

The proposed algorithm uses two components for feature detection. First, QRS complexes are detected by comparing the ECG signal's derivatives to an adaptive model of thresholds in a manner similar to [6] and [7]. P and T waves are detected using an innovative method based upon area calculations. The method of area calculations is much less susceptible to baseline wander and noise which may be present in ambulatory ECGs.

## 2 The QRS detector

The design goal for the QRS detector is to be as accurate as possible detecting the onsets, offsets, and peaks of QRS complexes. The QRS detector should be able to detect normal and abnormal ventricular activity such as PVCs, PACs, and pacemaker spikes. The QRS detector is also designed to require minimal computation and to operate on ECGs with various sampling rates and resolutions.

The QRS detector uses a digital filter to compute approximations to the ECG signal's first and second derivatives. Peaks in the second derivative are eval-

uated as candidate QRS peaks. The onsets and offset of each candidate QRS peak is determined using derivative information. Measurements for each QRS candidate is compared to an adaptive model of thresholds. If it satisfies the model, the candidate is accepted. The model thresholds are derived from the last accepted QRS candidate.

## 2.1 Computing Derivatives

Reddy et al. [4] present a 9 point central difference equation for computing the first derivative  $d_i$  from ECG samples  $x_i$  by the following formula:

$$\text{define } D(x_i) = \begin{pmatrix} -1/256 \\ -3/32 \\ -1/2 \\ -1 \\ 0 \\ 1 \\ 1/2 \\ 3/32 \\ 1/256 \end{pmatrix}^T \begin{pmatrix} x_{i-4} \\ x_{i-3} \\ x_{i-2} \\ x_{i-1} \\ x_i \\ x_{i+1} \\ x_{i+2} \\ x_{i+3} \\ x_{i+4} \end{pmatrix} \quad (1)$$

$$\text{then } d_i := D(x_i) \text{ and } d_i^2 := D(d_i^2) \quad (2)$$

This filter was found to have excellent properties for removing baseline wander and noise from the ECG while enhancing QRS complexes.

The filter presented by Reddy was modified in two ways to meet the goals of the QRS detector. First, the factor  $3/32$  was changed to  $1/16$  so that all coefficients of the filter are powers of two. This eliminates the need for integer divisions since microprocessors can perform multiplication by powers of two by register shifts. On an Intel Pentium processor, this improved performance by 28%. The modified filter also has excellent properties for enhancing QRS complexes while removing noise.

The performance of this filter depends on the sampling rate of the ECG. Figure 1 shows the frequency response of this filter. This filter was found to perform the best on ECGs with sampling rates of 125Hz. For other sampling rates  $F_s$ , the indices  $i$  of the samples  $x_i$  are modified by a factor  $\delta$  where  $\frac{\delta = F_s}{125Hz}$  to approximate resampling the ECG at 125Hz.

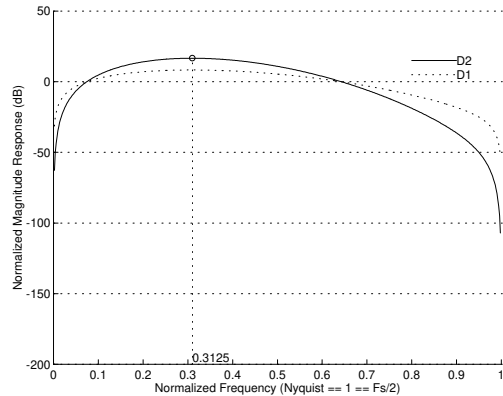


Figure 1: Frequency response of the modified  $D1$  and  $D2$  filters

The modified first derivative filter used by the QRS detector is given by the following formula.

$$D(x_i) = \begin{pmatrix} -1/256 \\ -1/16 \\ -1/2 \\ -1 \\ 0 \\ 1 \\ 1/2 \\ 1/16 \\ 1/256 \end{pmatrix}^T \begin{pmatrix} x_{i-4\delta} \\ x_{i-3\delta} \\ x_{i-2\delta} \\ x_{i-\delta} \\ x_i \\ x_{i+\delta} \\ x_{i+2\delta} \\ x_{i+3\delta} \\ x_{i+4\delta} \end{pmatrix} \quad (3)$$

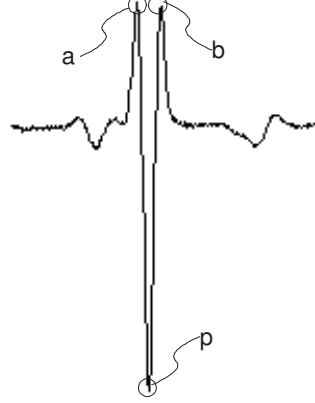


Figure 2: QRS peak detection in  $\{d_i^2\}$

Using this filter, the first and second derivative approximations  $d_i$  and  $d_i^2$  are derived from the ECG input  $x_i$  and are used by the QRS detector.

## 2.2 Locating the Peak of the QRS Complex

An adaptive model of thresholds and a record of the last candidate QRS is used to evaluate each sample in the ECG as a candidate QRS. The thresholds are adaptive in that they are derived from the last detected QRS complex, or set to initial values if no previous QRS was detected. In the case where no QRS is detected within a maximum period, the model thresholds are relaxed and the ECG signal is re-evaluated. This case is common in ECGs where there may be periods of high-amplitude QRS complexes followed by low-amplitude QRS complexes such as may be seen in paced rhythms. The thresholds used in QRS detection are given in the table below.

Threshold	Description	Initial Value
<i>D2Thresh</i>	Minimum QRS peak height in $d_i^2$	0
<i>MaxWidth</i>	Maximum QRS width	?
<i>D2Height</i>	Minimum vertical distance between QRS peak and onset in $d_i^2$	0
<i>MinHeight</i>	Minimum vertical distance between QRS peak and onset in $x_i$	0
<i>MinSep</i>	Minimum time between QRS complexes	??
<i>MaxSep</i>	Maximum time between QRS complexes	??

The QRS detector considers each sample  $x_p$  in turn and determines whether this point is or is not the peak of a QRS complex. Figure 2 is a typical plot of  $\{d_i^2\}$  centered about  $p$ . The points  $a$  and  $b$  are the indices of the local extrema in  $\{d_i^2\}$  adjacent to  $p$ . They are approximations to the onset and offset of the QRS complex with peak at  $p$ . The following tests are performed on these points:

1.  $d_p^2$  must be a local extrema in  $\{d_i^2\}$ .  $(d_p^2 - d_{p-1}^2) \times (d_p^2 - d_{p+1}^2) \geq 0$ .

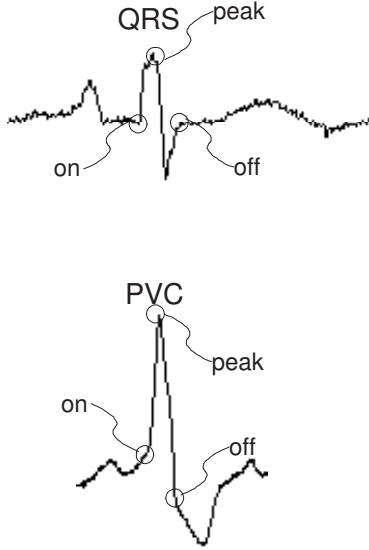


Figure 3: Other shapes of QRS complexes

2. The peak of  $dd_p$  must be above a threshold  $D2Thresh$ .  $|d_p^2| \geq D2Thresh$ .
3. The extrema cannot be too far apart together.  $b - a \leq MaxWidth$ .
4. The minimum distance from either extrema to the peak must be greater than a threshold  $D2Height$ .  $\min\{|d_p^2 - d_b^2|, |d_p^2 - d_a^2|\} \geq D2Height$ .

If a QRS has been previously detected, and  $p - LastQRS > MaxSep$  and the model thresholds are not at their initial value, then the model thresholds are relaxed, and the points between  $LastQRS$  and  $p$  are re-examined with the new model thresholds.

After the peak at  $p$  has satisfied the model thresholds, further refinement of  $a$  and  $b$  is necessary to correctly determine its onset and offset. The first approximations to a QRS complex's onset and offset  $a$  and  $b$  assume that the QRS complex is a single upright or inverted spike. QRS complexes may also be broad (such as PVCs) or contain smaller deflections between the onset or the Q and offset of the S waves

as shown in Figure 3. The region from the onset to the offset of the QRS complex should contain all points with high first and second derivatives. The following steps are used to determine the location of the QRS' onset and offset.

1. Compute a threshold for the first derivative between the peak  $p$  and the onset  $a$ . Set  $d_{max} := 1/4 \max_{a \leq i \leq p} \{|d_i|\}$ .
2. Compute a threshold for the second derivative between the peak  $p$  and the onset  $a$ . Set  $d_{max}^2 := 1/4 \max_{a \leq i \leq p} \{|d_i^2|\}$ .
3. Find the first index  $a'$  less than  $p$  where  $d_{a'}$  and  $d_{a'}^2$  fall below the thresholds  $d_{max}$  and  $d_{max}^2$ . Find  $a'$  such that
  - (a)  $|d_{a'}^2| \leq d_{max}^2$
  - (b)  $|d_{a'}| \leq d_{max}$
  - (c)  $a' < p$
  - (d)  $|p - a'|$  is minimal.
4. Repeat the previous steps to find  $b' > p$  with  $d_{max} := \max_{p \leq i \leq b} \{|d_i|\}$ .

If  $b' - a' < MaxWidth$  then they are recorded as the onset and offset of the QRS with peak at  $p$  and  $a$  is set to  $a'$  and  $b$  is set to  $b'$ . Otherwise, the original indices  $a$  and  $b$  are used.

If all the conditions are met, the point at  $p$  is a valid candidate for a QRS peak, and  $a$  and  $b$  are the QRS' onset and offset. QRS complexes cannot occur arbitrarily close together. If the previously accepted QRS candidate  $LastQRS$  also passed the above tests, then  $p - LastQRS$  must either be greater than a minimum separation,  $MinSep$ , or one of the two must not be a QRS. If the two points are greater than  $MaxSep$  samples apart, then the point  $LastQRS$  is accepted as a QRS and output along with its onset and offset. The index  $p$  is then recorded as the last QRS candidate. If the two points are too close together, the one with the least absolute value in  $\{d_i^2\}$  is rejected.

If the QRS at  $p$  is accepted, the model of thresholds is derived from it as follows:

Figure 4: P and T waves adjacent to a QRS

Threshold	New Value
$D2Thresh$	$1/4 d_p^2 $
$MaxWidth$	$2(b - a)$
$D2Height$	$1/4 \min \{ d_p^2 - d_b^2 ,  d_p^2 - d_a^2 \}$
$MinHeight$	$1/4 \min \{ x_p - x_b ,  x_p - x_a \}$
$MinSep$	??
$MaxSep$	$2(p - LastQRS)$

### 3 The P and T Wave Detector

Once a sequence of QRS complexes are detected, a separate algorithm is used to detect P and T waves in the intervals before and after each QRS complex. Within each interval, an algorithm is used to choose the best onset, offset and peak of a P or T wave present in that interval. Figure 4 shows a typical ECG signal with P and T waves marked.

P and T waves are shallow, rounded features unlike QRS complexes which are tall and sharp. The P wave is especially shallow and is often obscured by noise and baseline wander. The derivatives of P and T waves may not be significantly greater than those of baseline wander and noise present in the ECG. This property makes algorithms based on derivative calculations such as the one used by the QRS detector unreliable.

The proposed algorithm uses a filter to remove noise from the ECG and area calculations to determine the onset and offset of P and T waves. The following steps are performed:

1. Filter all samples in the interval to remove noise.
2. Consider each pair  $(a, b)$  of indices in the interval as a candidate for the onset and offset of a P or T wave.
3. Reject  $(a, b)$  if they are too close together or too far apart.  $MinWidth \leq |b - a| \leq MaxWidth$ .
4. Find the index of the peak  $p$  between  $a$  and  $b$  and its height  $h$ . The peak is the point with maximal vertical displacement from the line from the

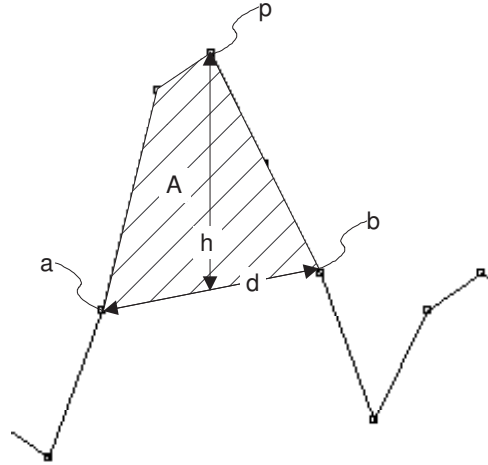


Figure 5: Measurements used to compute the score of a potential P or T wave

point  $(a, x_a)$  to  $(b, x_b)$ .  $p$  and  $h$  are given by the following equation:

$$h = \max_{a < p < b} \left| x_p - \left( x_a + (p - a) \frac{x_b - x_a}{b - a} \right) \right| \quad (4)$$

5. Reject  $(a, b)$  if the peak is too low.  $h < MinHeight$ .
  6. Calculate the area  $A$  of the closed polygon bounded by  $(a, x_a), (a + 1, x_{a+1}), \dots, (b, x_b)$
  7. Calculate the “score”,  $\gamma$ , of the pair  $(a, b)$  as
- $$\gamma = \frac{h|A|}{d} \quad (5)$$
8. The pair  $(a, b)$  and its peak  $p$  which has the maximim score  $\gamma$  is output as the onset, offset and peak of the P or T wave in the interval.

#### 3.1 Calculating the area of a closed polygon

A formula for computing the area bounded by a closed polygon with at least three vertices  $(t_i, x_i)$  is given by:

$$A_{n-1} = \frac{1}{2} \left( t_0 x_{n-1} - t_{n-1} x_0 + \sum_{i=0}^{n-2} t_{i+1} x_i - t_i x_{i+1} \right) \quad (6)$$

Formula 6 is derived from an elegant method for computing the area by arranging all the vertices of a polygon in a table, cross-multiplying entries, and summing the cross products as shown below:

$$\begin{array}{ccccccc}
 & & t_0 & & x_0 & & \\
 t_1 x_0 & \leftarrow & t_1 & \times & x_1 & \rightarrow & t_0 x_1 \\
 t_2 x_1 & \leftarrow & t_2 & \times & x_2 & \rightarrow & t_1 x_2 \\
 \vdots & & \vdots & & \vdots & & \vdots \\
 t_{n-1} x_{n-2} & \leftarrow & t_{n-1} & \times & x_{n-1} & \rightarrow & t_{n-2} x_{n-1} \\
 t_0 x_{n-1} & \leftarrow & t_0 & \times & x_0 & \rightarrow & t_{n-1} x_0 \\
 \hline
 & & A_0 & & & & A_1
 \end{array} \quad (7)$$

The area of the polygon is derived from the sums  $A_0$  and  $A_1$  like so:

$$A = \frac{A_0 - A_1}{2} \quad (8)$$

The sequence of ECG samples  $\{x_i\}$  is regularly spaced. A subsequence of samples  $\{x_i\}_{i=a}^b$  then defines a polygon with vertices at  $(a, x_a), (a+1, x_{a+1}), \dots, (b, x_b)$ . The area under this subsequence of samples can be calculated as above.

To reduce computation, area calculations can be reused. If  $A_{a,b-1}$  is the area under the subsequence of samples  $\{x_i\}_{i=a}^{b-1}$ , then the area under the polygon which includes the succeeding sample  $x_b$  can be computed as:

$$A_{a,b} = A_{a,b-1} + (x_{b-1} - x_a) - (b-a)(x_b - x_{b-1}) \quad (9)$$

### 3.2 Wavelet Filtering

The proposed algorithm uses wavelet filtering [1] to remove noise from the ECG signal before detecting P and T waves. Wavelet filters were found to be better than more traditional FIR filters at removing noise without blurring the corners at the onsets and offsets

of P and T waves. Wavelet filtering is performed on the intervals before and after each QRS complex where P and T waves are to be detected.

Wavelet filtering is accomplished by computing the DAUB4 coefficients of a region, zeroing all coefficients in the lowest two scales, and taking the inverse transform.

## 4 Results

The P, QRS, T detection algorithm was tested at Calgary General Hospital to determine its accuracy. Tests were performed on a subset of ECG data from the MIT database. An operator viewed these ECGs using Harley Street Software's WAM product and counted the number of P, QRS, and T waves that were present in the ECG and the number of correct and incorrect wave annotations that the P, QRS, T detector produced. Incorrect waves were defined to be waves for which the either the onset, offset, or peak of the wave were incorrectly placed by the P, QRS, T detector. The data set derived from the tests is presented below. Accuracy is measured as the ratio of correct to total waves. The number following the ECG record number in the first column is the lead number for that record.

On the whole, P, QRS and T wave detection was found to very accurate with average accuracies over over 98% for all three types of feature.

The worst P wave detection was 90.39% in record 111. This is due to the low-amplitude biphasic P waves in the second channel of that record.

The worst T wave detection was 93.72% in record 112. This was due to very low-amplitude T waves in the second channel of that record.

MIT ECG		P			QRS			T		
Rec	Dur	Tot	Corr	Miss	Tot	Corr	Miss	Tot	Corr	Miss
100 1&2	2:00	147	145	3	147	145	0	147	147	0
101 1	2:00	124	121	3	124	122	2	124	102	22
103 1&2	2:00	141	140	1	141	141	0	141	141	0
105 1	3:00	243	242	1	243	240	1	241	241	2
111 1&2	10:01	687	621	66	687	682	5	687	687	0
112 1&2	10:01	892	885	7	892	885	7	892	839	53
113 1&2	10:01	573	573	0	573	573	0	573	573	0
115 1&2	10:01	635	635	0	635	635	0	635	635	0
116 1&2	10:01	755	755	0	755	755	0	755	755	0
117 1&2	10:01	478	478	0	478	478	0	478	478	0
119 1&2	10:01	515	515	0	515	515	0	515	515	0
202 1	10:00	267	260	7	267	260	0	267	267	0
Totals	87:07	5457	5370	88	5457	5431	15	5455	5380	77
Accuracy		98.41%			99.52%			98.63%		

Figure 6: Testing results

## 5 Limitations

The proposed algorithm relies on QRS detection to detect regions in which to identify P and T waves. Thus, it is unsuitable for detecting atrial activity in the presence of atrial flutter, or other atrial rhythms which are dissociated from the ventricular.

P and T waves are assumed to be either upright or inverted humps. The onsets and offsets of biphasic P or T waves may not be detected correctly.

The algorithm used to detect P and T waves scores every pair of points within an interval to determine which points best determine the onset and offset of the wave. Such an algorithm inherently requires on the order of  $n^2$  operations, where  $n$  of the number of samples in the interval. This is much more computationally expensive than the QRS detector detailed above.

## Conclusion

An algorithm for detecting P, QRS, and T waves in a single lead ECG is presented. This algorithm can accurately detect these waves in the presence of baseline wander and noise in ECGs with a variety of sampling rates.

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